EFFICIENT IRIS RECOGNITION USING REDUCED IRIS CODE METHOD

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COMMITTEE DECISION

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DEDICATIONS

To the person who taught me the moral of human life... To the source of strength, and the tender affection... To the source of glory and pride... To my grandfather, Saleh H. Al-Omari

To my inspiring in life, the symbol of challenge and hard working... The one who taught me determination and persistence... The one who granted me the courage and the strength, the truth and the generosity... To my father, my soul Hasan S. Al-Omari..

To the resource of peace and sympathy... To the brightness in the darkness... To the one who granted me love without tiredness.. To the one who prays for me every night... To the pure heart... My darling mother, Sameeha M. Al-Omari...

To the candles that enlightened my life with hope... To the one who covered me with love... The one who always encouraged and supported me without waiting awardnessTo my dearest uncles... Hussien S. Al-Omari & Mahmoud S. Al-Omari

To the rosy part of my life, my dearest brothers, sisters & cousins, who always supported me with their love and compassion.

To those who lived the experience with me... Who helped me make the difficult easy.. Who stood beside me all the times with their love, advice and supportTo my friends especially Ruba, Shatha & Oraeb

To all those who stood by my side and gave me moral support to finish this work



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LIST OF ABBREVIATIONS

ABBREVIATION	MEANING
HD	Hamming Distance
DCT	Discrete Cosine Transform
POS	Product Of Sum
WED	Weighted Euclidean Distance
НСР	Hamming Center Problem
ABI	The new proposed model takes its name from the A, B and I arrays that is generated during the human iris code representation stage.
C factor	Comparison factor
Pdb	Pointers of the database iris code
Pin	Pointers of the input iris code



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ABSTRACT

The science of identifying individuals based on their physiological and behavioral characteristics is evolved rapidly these days. Iris recognition is a method of biometric authentication that uses pattern recognition techniques based on high-resolution images of the irises of individual's eyes. This method is interesting especially when there is a need to search very large databases without acquiring any false matches even with a huge number of possibilities.

But the problem with the exhaustive methods appears when searching large datasets of iris codes. The experiments showed that applying comparisons between 100,000 iris codes require 1 second; and whenever the dataset becomes larger the time of comparison operation will increase. So if we have 100,000,000 iris codes the comparison operation will take around 1000 second which is not efficient at all.



In this study, an efficient alternative for human iris code retrieval is presented. The proposed model is mainly based on representing the human iris binary codes which enhanced the matching and searching operations.

The iris codes representation operation looks for iris code bits as a number of pairs, each pair is represented by one bit using a three one dimensional arrays A, B and I. The array A is the master array that contains the representation of the identical pairs while the array B, the secondary one, contains the representation of the different pairs. Pointers go from the master array to the secondary array when meeting a different pair during the representation operation, the pointers indexes are stored in the third array I.

The improvement that was provided by the proposed ABI model was proven by applying a mathematical analysis for the standard and the ABI techniques. The analysis showed that the ABI model complexity is linear while the standard model complexity is quadratic. Because when applying the ABI model, the iris code 2048 bits are visited only once (at the beginning of the whole matching operation, during the code representation step), while using the standard model the iris code 2048 bits are visited each time a match operation is applied between the input iris code and every iris code in the database.



1. INTRODUCTION



1. Introduction

The human iris pattern is considered the most reliable, secure and accurate because it becomes stable after one year of birth and can not be easily damaged and imitated. Moreover it is more complex than other physiological and behavioral characteristics such as face, fingerprints, eye retina, signature and voice. (Daugman, 1993), (Wildes, 1997), (Jain et al., 1999), (Mansfield and Wayman, 2002), and (Daugman, 2003).

The iris is the circular, colored curtain which is located between the pupil and the white sclera in the eye. Each person has a unique texture of the iris that can distinguish his/her signature from other's.

In order to apply the iris recognition technique we have to capture a rich-detailed iris image (Image Acquisition). Locate the inner and outer boundaries of the iris (Iris Localization). Map the iris region to a rectangular one (Iris Normalization). Finally represent the iris pattern as a binary one (feature extraction (encoding)), at this stage we can apply the pattern matching.

1.1 Problem Statement

When we use the iris pattern for security issues, time is very essential. There are two main factors that affect the time of iris code retrieval from the dataset. Firstly is the size of



the iris codes dataset and the search method that we apply over it to retrieve the needed code. Secondly, the size of the iris code it self.

The existing iris recognition technique using binary (XOR) operations to calculate the hamming distance between the different matched pair of iris codes. The required time to apply one XOR operation is around 10μ s, (Daugman, 2004), which means that the time to recognize one binary code out of 100,000,000 dataset is around 1000 second. So if we reduce the number of the (XOR) operations that are applied over the iris codes bits an enhancement will occur.

1.2 Objectives

This work aspires to meet a number of objectives that are summarized as follows:

- Reduce the overhead of the existing iris codes recognition technique.
- Apply the ABI Model (the proposed model that takes its name from the (A, B & I) arrays that are used to represent the human iris binary codes).
- Apply comparison between the existing approach (Daugman, 2004) and the proposed ABI model.

1.3 Contributions

In this study we proposed a model that reduces the overhead of the exhaustive approach by focusing on the main two factors that affect the time of iris code retrieval



operation. Firstly it reduces the number of bits to be compared when applying the match operation between the binary iris codes. Secondly it reduces the domain of the search operation.

The role of the proposed ABI model started after the iris code encoding stage of the human iris recognition technique (this model deals with the iris binary codes). The ABI Enhanced the pattern matching stage by representing the binary iris codes in a way that gives the ability to decide the similarity of the iris codes without the need to compare all the codes bits. Also it reduced the number of the dataset iris codes that will be compared with the input iris code.

1.4 Thesis organization

This thesis contains four chapters outlined as follows:

Chapter one: literature review that discuss the human iris recognition technique starting from the image capturing operation and ending with pattern matching.

Chapter two: theory and implementation that mention a brief description to the basic concepts that we will refer to in our work. And then discuss in details the enhanced human iris code retrieval using our ABI model.

Chapter three: discussion and mathematical analysis that present an evaluation for the ABI model. The evaluation is based on the mathematical analysis, models complexity and the



test cases that present the headlines of all the cases that maybe appear during the human iris code recognition technique in both the standard and the ABI methods.

Chapter four: conclusions of the thesis and future work.



2. LITRETURE REVIEW



2. Literature Review

In this chapter we introduced some basic concepts that are relate to this thesis. First an overview is given about the human iris components and properties. Then the iris recognition system phases are briefly described. Also a survey of existing techniques and algorithms for the iris recognition system is mentioned.

2.1 Human iris code recognition

The iris is the circular, colored curtain which is located between the pupil and the white sclera in the eye. Each person has a unique texture of the iris that can distinguish his/her signature from other's, Figure 2.1 shows the human eye components.



Figure 2.1: Human eye components.

Flom and Safir noticed the human iris stability and unique texture over human life time and suggested to use the human iris as the basis of biometric (Flom and Safir, 1987).



The iris recognition fundamentals algorithms were developed in 1989 by John Daugman. These algorithms were patented by Daugman (Daugman, 1993), (Daugman, 1994) and (Daugman, 2004) which then becomes the basis for all the iris recognition systems. In order to apply the iris recognition technique we have to follow 5 stages as described in Figure 2.2.



Figure 2.2: The iris recognition technique stages.

2.1.1 Capture a rich-detailed iris image (Image Acquisition):

Capturing iris images with high quality is one of the main issues of iris recognition system, for this the use of specifically designed sensors and light sensitive cameras is essential. Figure 2.3 presents how to capture a rich detailed iris image in the image acquisition stage.





Figure 2.3: Capturing a rich detailed iris image

To get an acceptable iris images the captured images should be centered with good contrast in the interior iris patterns and do not forget the sufficient resolution and sharpness (Wildes, 1997). Figure 2.4 shows the problems that will appear in segmentation (Localization) stage when the human iris image is captured in a wrong way.



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Figure 2.4: The problems that will appear when the human iris image is captured in a wrong way.

2.1.2 Locate the inner and outer boundaries of the iris (Iris Localization):

Locating and detecting the iris inner and outer boundaries (pupil and limbus) is an important process that affects the accuracy of the matching results, the more accurate iris localization and segmentation leads to more accurate matching results.



Daugman's systems (Daugman, 1988) based on integro-differential operator for iris localization; the integro-differential operator is defined in Equation (1).

$$\max_{(r,x_0,y_0)} \left| G_{\sigma}(r)^* \frac{\partial}{\partial r} \oint_{r,x_0,y_0} \frac{I(x,y)}{2\pi r} ds \right|$$
(1)

Where I(x, y) is the eye image, r is the radius, the symbol * denotes convolution and $G\sigma(r)$ is a smoothing function such as a Gaussian of scale σ . ds is an element of a circular arc defined by radius r and a center (x₀, y₀).

This operation behaves as a circular edge detector, blurred at a scale set σ , which searches iteratively for a maximum contour integral derivative with increasing radius. Defining a path of contour integration through the three parameter space of center coordinates and radius (x₀; y₀; r) (Daugmn,2001). Figure 2.5 shows the iris and pupil localization and eyelids detection using Daugman's algorithm (Daugman, 2004).





Figure 2.5: The iris and pupil localization and eyelids detection using Daugman's algorithm (Daugman, 2004).

Such works have always aimed at improving the iris localization algorithms (Cui et al., 2004), (Kong and Zhang, 2001), (Tisse et al., 2002), (Huang et al., 2004), (Maenpaa, 2005).

2.1.3 Map the iris region to a rectangular one (Iris Normalization):

Iris Normalization is the process of recognizing the human iris codes regardless of the size, position and rotation and also removing the dimensional contradictions between irises.

Daugman represents each point within the iris region to a pair of polar coordinates (r,θ) where r is on the interval [0,1] and θ is angle $[0,2\pi]$ in what called homogenous rubber sheet model (Daugman, 2001).



$$I(x(r,\theta), y(r,\theta)) \rightarrow I(r,\theta)$$
 (2)

Where

$$x(r,\theta) = (1-r)x_{p}(\theta) + rx_{s}(\theta)$$
(3)

$$y(r,\theta) = (1-r)y_{p}(\theta) + ry_{s}(\theta)$$
(4)

where I(x,y) is the iris region image, (x,y) are the original Cartesian coordinates, (r, θ) are the corresponding normalized polar coordinates, (x_p(θ),y_p(θ)) are the pupil boundary points along the θ direction, and (x_s(θ),y_s(θ)) are the iris boundaries coordinates along the θ direction. This representation takes into account the pupil dilation and the inconsistencies in the iris size; this process is graphically discussed in Figure 2.6.





Figure 2.6: Daugman's Rubber Sheet Model (Masek and Kovesi, 2003).

Figure 2.7 presents the output of the normalization operation for a segmented iris image, the normalized iris and the noise mask.



Figure 2.7: The normalization operation for the segmented iris image.



2.1.4 Feature extraction (Encoding):

This process seeks to extract the featured iris patterns data in order to generate and build templates so that the comparisons between the templates can be done.

Daugman employed two dimensional Gabor filters to extract the texture of an iris. The gabor filter is constructed by modulating a sin/cosine wave with a Gaussian to achieve optimal joint localization in space and spatial frequency (Daugman,1993). A 2D Gabor filter over the image domain (x, y) is presented in Equation (5).

$$G(x, y) = e^{-\pi \left[\frac{(x-x_0)^2}{\alpha^2} + \frac{(y-y_0)^2}{\beta^2}\right]} e^{-2\pi \left[u_0(x-x_0) + v_0(y-y_0)\right]}$$
(5)

Where (x_0, y_0) specify the position in the image, (α, β) specify the effective width and length, and (u_0, v_0) specify modulation, which has the spatial frequency $\omega_0 = \sqrt{u_0^2 + v_0^2}$.

2.1.5 Measure the similarity between two iris patterns (Pattern matching):

The last stage of the iris recognition is to measure the similarity between two iris patterns and then give the result to identify or deny the person. The hamming distance is one of the simplest matching metrics that was proposed by Daugman to measure the similarity of the iris patterns bits (Daugman, 1993). The higher the hamming distance is,



the more different the iris patterns are. The Hamming Distance (HD) is defined as the sum of exclusive –OR between X and Y as illustrated in Equation (6).

$$HD = \frac{1}{N} \sum_{j=1}^{N} \mathbf{X}_{j} \bigotimes \mathbf{Y}_{j}$$
 (6)

Where X and Y are the two iris patterns and N is the total number of bits in the encoding iris patterns.

After calculating the HD for 9.1 million iris codes that was related to different persons, it was found out that it is probable that two different irises might agree at most a third of their bits. The results were the same when the previous measurement was applied over the genetically identical irises and also for the irises of identical twins (Daugman, 2004). All these measurements showed that in order to distinguish people by their iris patterns the degree of match should be (HD ≤ 0.32).

In Equation (7) the noise masking is including to calculate the hamming distance between two iris patterns (Daugman, 2004). Since each bit in any iris code may be 1 or 0, the hamming distance between two independent iris codes would be 0.5.

$$HD = \frac{\|(codeX \otimes codeY) \cap maskX \cap maskY\|}{\|maskX \cap maskY\|}$$
(7)



Where codeX and codeY are the iris patterns to be compared, maskX and maskY are the corresponding noise masks for codeX and codeY.

Daugman used Equation (8) to calculate the false match probabilities for various HD criteria (Daugman, 2004), the calculation results are illustrated in Table1.

$$n! \approx \exp(n\ln(n) - n + \frac{1}{2}\ln(2\Pi n)) \tag{8}$$

HD criterion	False match probability
0.26	1 in 1013
0.27	1 in 1012
0.28	1 in 1011
0.29	1 in 13 billion
0.3	1 in 1.5 billion
0.31	1 in 185 million
0.32	1 in 26 million
0.33	1 in 4 million
0.34	1 in 690,000
0.35	1 in 133,000
0.36	1 in 28,000
0.37	1 in 6750
0.38	1 in 1780

Table 1: Single false match probability for various HD criteria (Daugman, 2004).

The one dimensional Discrete Cosine Transform (DCT) was used to produce the iris codes (Monro et al., 2007), who then used product of sum (POS) of an individual sub feature hamming distance to match between the iris codes as defined in Equation (9).



$$HD = \left(\prod_{i=1}^{M} \frac{\sum_{j=1}^{N} (SubFeature 1_{ij} \otimes SubFeature 2_{ij})}{N}\right)^{\frac{1}{M}}$$
(9)

M x N is considered the size of the iris code, M is the number of bits per sub feature, and N is the total number of sub features in a feature vector. The sub feature bits are XORed, and the resultant vector is summed and normalized by dividing by N. This approach is done for all M sub features. The hamming distance value will be in the range from 0 to 1.

Ma et al. (2003) proposed a simple and effective algorithm for iris recognition in which the iris code matching was achieved using XOR operation.

The Weighted Euclidean Distance (WED) between the corresponding feature vectors was used by (Zhu et al., 2000) to measure the similarity of the iris patterns and apply the pattern matching, the WED can be calculated using Equation (10).

$$WED(k) = \sum_{i=1}^{N} \frac{(f_i - f_i(k))^2}{(\delta_i(k))^2}$$
(10)

Where f_i is the ith feature of the unknown iris, and $f_i(k)$ is the ith feature of the iris pattern k, and $\delta_i(k)$ is the standard deviation of the ith feature in the iris pattern k.



Boles (1998) used zero crossing 1D wavelet to extract the iris image features, represented them using one dimensional (1D) signals and then compared the resulted signals with the model's features via different dissimilarity functions.

An evaluation of the matching degrees and similarity via a normalized relation between the acquired and the existing database representations was applied by (Wildes, 1997).

2.2 Approximation Algorithms for the Hamming Center Problem (HCP)

The randomized rounding was used to achieve an approximate solution to Hamming Center Problem which is close to the optimum one with high probability when the radius is large compared with the input size, (Gramm et al., 2001). The problem with this solution is that it gives poor approximations in the cases where the radius is small.

A randomized $(4/3 + \epsilon)$ approximation algorithms for the HCP, where ϵ can be set to any constant > 0, was introduced by (Gasieniec et al., 1999); if the radius equals O (1) this algorithm runs in a polynomial time, otherwise the algorithm will be exponential to the size of the input.



2.3 Human iris code retrieval from large dataset

In the traditional iris code retrieval method, each bit in the first iris code is compared with the corresponding bit in the second iris code using binary Exclusive-OR, and then the resulted value used to calculate the HD.

The required time to apply XOR comparison between two iris codes is around $10\mu s$, (Daugman, 2004), which means that the time to recognize one binary code out of 100,000,000 dataset is around 1000 second.

If the calculated HD value exceeds the specified threshold (HD \leq .32) then they decide that the given codes related to two different persons, if not, they relate to the same person (Daugman, 2004).

The existing iris code retrieval method use a linear search operation to find the needed iris code in the dataset, which is efficient for small datasets, but whenever the dataset becomes larger the time of search operation will increase.



3. THEORY AND IMPLEMENTATION



3. Theory and Implementation

At the beginning of this chapter we mentioned a brief description to the basic concepts that we will depend on in our work, these concepts are the HD and HCP, the Test of Statistical Independence and also the Decision Environment for Iris Recognition system.

Then we discussed in details the enhanced human iris code retrieval using the ABI model which consists of several phases. Firstly the human iris code representation. Secondly the pattern matching which will be applied in two stages, the ABI initial matching stage and the ABI main matching stage. Finally the Search criterion over the dataset using the ABI model will be discussed.

3.1 Hamming Distance (HD)

The hamming distance concerns about the conversion of one string into another by fetching the number of different bits between two binary strings which gives an indication to the minimum number of bits that must be substituted to complete the conversion operation. Named after the mathematician "Richard Hamming" (Hamming, 1950).

We mentioned the following examples to clarify the idea of the HD:

101110100101 and 100100101101	➔ Differences = 3.
214387659 and 223379654	➔ Differences = 5.
"Sort" and "Port"	➔ Differences = 1.
المنسارات	

3.2 Hamming Center Problem (HCP)

The HCP is the problem of finding a binary string of length n (not necessarily in S (set of binary strings)) which is close to each string in S, it is also known as "the minimum radius problem" in (Norton and Salagean, 1999) and "the closest string problem" in (Gasieniec et al., 1997) and was classified as NP –hard by (Frances and Litman, 1997).

3.3 The Test of Statistical Independence

The test of statistical independence is implemented using the Hamming Distance measurement (HD). The key factor to the success of the iris recognition systems is to fail the test of statistical independence. The failure of the test can be achieved if any iris pair produces a hamming distance less than the predefined threshold value and thus the two iris codes refer to the same person.

3.4 Decision Environment for Iris Recognition system

Decidability is the process of recognizing persons using their iris patterns which can be obtained by comparing the HD distributions for same and different irises (Daugman, 2004).



Figure 3.1 presents the distribution of HDs for different pairs of same-eye images (in left side) and for different pairs of different eyes (in right side). Such distribution called the decision environment and the overlap between the two distributions represents the error rates.



Figure 3.1: The Decision Environment for iris recognition (Daugman, 2004).

Figure 3.1 shows that more than half of same human iris codes have a HD value equals to 0.00, with a mean value equals to 0.019. In contrast, the different human iris codes have an average HD value equals to 0.456.


3.5 Enhanced iris code retrieval using ABI Model

The iris recognition is a fast, accurate and secure biometric technique that offers the highest accuracy in verifying and identifying individuals. This is because no two irises are alike, not between identical twins, or even between the left and right eye of the same person. Iris recognition technology addresses the problems of both password management and fraud.

For these properties this technology becomes widely used at physical access points that require high security, such as airports, government buildings, and research laboratories. The future vision for iris recognition technology is to replace most current forms of physical access-based identification such as password, personal identification number (PIN) or key.

So the need for iris recognition technology enhancement becomes a must, as much as we enhance this technology the admired future vision will be reached faster. In order to enhance the traditional human iris code retrieval from the iris database, we established the ABI model.

The ABI model firstly represents the iris codes and then applies the codes matching in two stages initial and main matching stages. Finally the search criteria over the dataset will be enhanced using the ABI model.



3.5.1 Iris code representation using ABI model

The main idea that the ABI model based on to enhance the existing human iris code retrieval from large dataset is to represent the iris codes inside the dataset, this operation will represent each consecutive pair in the iris code by one bit.

To do so we used three one dimensional arrays (A, B and I) such that the array A is the master array that represents the identical bit pairs (00, 11). While the array B is the secondary array that represents the different bit pairs (01, 10) and I is the array that contains the pointers indices that goes from the array A to the array B. Figure 3.2 shows the ABI model iris codes representation flow chart.





Figure 3.2: Iris code representation using the ABI model flow chart.



1- Initialization phase

The iris code representation algorithm initializes number of parameters. Table 2 illustrates these parameters, first it defines the A, B and I arrays then starts to apply the representation of the input iris code using them aside with the r, c and p that act as counters which values increase each time we move to the next bit in the original iris code.

Description	Parameter
The primary array with length of original iris code / 2	
that will represent the identical pairs of the original iris	
code such as 00 or 11, or it will contain a pointer to the	А
corresponding index in array B if the original iris code	
pair was 01 or 10	
The secondary array with length of original iris code / 2	
that will represent the different pairs of the original iris	В
code such as 01 or 10	
This array will contain the index (location) of pointers	T
that goes from array A to array B	1
Counter that will be increased each time that we will	
represent a pair of the original iris code and will act as	r
array A and array B index	
Counter that will visit each hit of the original iris code	с
Counter that will be increased each time we access the I	
array and also it represents the number of pointers that	р
goes from array A to array B plus 1	

Table 2: The parameters that configured by iris code representation algorithm.



2- Iris code representation phase

In this phase the binary iris code that was resulted from the encoding stage of the iris recognition technique with length of 2048 bits will be represented using three one dimensional arrays (A, B and I) with length of 1024. We will represent every two consecutive bits in the original iris code in one bit either in the array A or in the array B; this technique will be applied as follows:

- 1- If the bits in the original code were identical (00 or 11) then we will represent them as one bit in the primary array A (00 \rightarrow 0, 11 \rightarrow 1).
- 2- If the bits were different (01 or 10) then a pointer will be assigned from array A to array B and a one bit in the array B will represent these two consecutive bits (01 → 0, 10 → 1).
- 3- If step 2 was applied then the index of the pointer must be inserted in array I (eg: if the third and the forth bits in original code were 01 then a pointer will go from A[3] to B[3], and a 0 will be assigned to B[3] to represent the bits and finally the index of the pointer (3) will be assigned to array I).
- 4- These steps will be repeated until the whole iris code being represented.

As an example of how to represent the iris codes using the ABI model, in Figure 3.3 we apply codes representation using the ABI model over a 12-bit codes.





Figure 3.3: Applying the ABI model to represent the human iris codes.

The output for this phase is the arrays A, B and I and also the number of pointers (P) from the array A to the array B that equals to the length of the array I.

3.5.2 Iris patterns matching using ABI model

In the ABI model we will benefit from the first stage (the iris pattern representation) outputs (A, B, I and P), using these outputs will help us to apply the pattern matching in an efficient way.

Using the ABI model in pattern matching operation will avoid matching all the codes bits to decide whether the matched codes are different or relate to the same person, such that there will be two stages for the iris patterns matching; the initial matching stage and the main matching stage.



If the two iris patterns are completely different then using the ABI model in pattern matching operation will allow us to decide that they are related to different persons at the initial stage without the need to apply the bit by bit matching operation. The failure to distinguish the iris patterns at the initial stage will lead to a transition to the ABI main matching stage.

Figure 3.4 shows the iris codes matching criterion using our ABI model flow chart.





Figure 3.4: The iris codes matching criteria using the ABI model flow chart.



1- Initialization stage

The ABI iris codes matching algorithm initializes number of parameters. Table 3 presents a description for these parameters.

Description	Parameter
The number of pointers that differs between the two iris code	Difference
Initial Hamming distance	IHD
Comparison factor	С
An array of length iris code / 4 (array A / 2) that will contain the second half of array A of the first iris pattern	X1
An array of length iris code / 4 (array A / 2) that will contain the second half of array A of the Second iris pattern	X2
An array that will contain the second half of array I of the first iris pattern	S1
An array that will contain the second half of array I of the second iris pattern	S2
Differences between the first half of array A between the two iris codes that results via bit by bit matching	dif1
Differences in the second half of array A between the two iris code that can be calculated using arrays S1 and S2	dif2
The summation of dif1 and dif2	Total dif

Table 3: The parameters that configured by the ABI iris codes matching algorithm.



2- ABI initial matching stage:

In this stage we can benefit from the number of pointers from array A to array B (the number of the elements of the I array (P)), that we have got after applying the ABI iris codes representation algorithm, by taking the difference of the pointers between the two codes which will allow us to calculate the Initial Hamming Distance.

If the Initial Hamming Distance exceeds the allowed threshold (HD \ge 0.32), then we can decide that the two iris codes are not related to the same person, without the need to compare each bit in the iris codes. If the Initial Hamming Distance does not exceed the allowed threshold (HD \le 0.32), then we have to move to the second stage (Main matching operation).

- How to calculate the difference between the pointers?

To calculate the difference of the pointers between the needed iris codes we will use the array I, which contains the pointers locations (index) in the array A. If we take the example in Figure 3.3 where $II = \{2,4\}$ and $I2 = \{4\}$, here the difference = 1, the following are examples on how to calculate the difference:

 $I1 = \{1, 3, 4, 5\}, I2 = \{0, 2\} \rightarrow \text{the difference} = 6.$

 $I1 = \{2, 6\}, I2 = \{3, 5, 6\}$ \Rightarrow the difference = 3.



We can calculate the difference using Equation (11).

$$Difference = (I1 \cup I2) - (I1 \cap I2) \tag{11}$$

3- ABI main matching stage

In this stage we benefit from the Initial HD to calculate what we called a Comparison factor (C factor), in order not to match all the bits of the iris codes as in the traditional method.

- How to calculate the C factor:

We can define the C factor as the maximum number of differences that are allowed to be found between the two iris codes such that when calculate the HD, the result will not exceed the allowed threshold 0.32. So we can calculate the C factor as in Equation (12)

$$C = N * 0.32$$
 (12)

Where C is the Comparison factor and N is the total number of iris code bits.



- How to apply the ABI main matching stage?

In this stage we will take the sealing of (length of Array A) / 2 and then refer to the array I to find how many differences are there in the second half of the array A, and then apply the bit by bit comparison operation over the first half of A.

When the bit by bit comparison applied over the first half we will check:

- 1- If the number of differences in the first half plus the specified differences in the second half exceeds the C factor then the iris codes are not for the same person.
- 2- If the number of differences in the first half plus the specified differences in the second half does not exceed the C factor, then go to **3**.
- 3- Check if the number of differences plus all the remained bits are less than or equal the C factor value or not:

- If number of differences plus the remained bits $\leq C$ factor, then the two iris codes relate to the same person.

- If number of differences plus the remained bits > C factor then we have to repeat the previous steps. Until either the two codes are for different persons or the specified differences plus all the remained bits do not exceed the C factor which mean that the two iris codes relate to the same person.



The bit by bit matching operation is illustrated in Table 4.

First	Code	Second	d Code	Action					
Array A	Array B	Array A	Array B						
0	-	0	-	The same					
0	-	1	-	different					
1	-	0	-	different					
1	-	1	-	The same					
Pointer	0	Pointer	0	The same					
Pointer	0	Pointer	1	different					
Pointer	1	Pointer	0	different					
Pointer	1	Pointer	1	The same					
pointer	0 or 1	0 or 1	-	They are different without					
				referring to array B					
0 or 1	-	pointer	0 or 1	They are different without					
				referring to array B					

Table 4: The bit by bit matching operation in the ABI model.

Note that when applying the bit by bit matching it is not necessary to access the array B such that when array A of the first pattern contains a 0 or 1 and the corresponding place in array A of the second pattern contains a pointer then the ABI will record a difference without access the array B of the second pattern as discussed in Table 4.

Applying the ABI model over the human iris codes will change the iris recognition system stages such that the binary iris code representation stage, the initial matching stage and the main matching stage will be added as shown in Figure 3.5.





Figure 3.5: the iris recognition system stages with the ABI model.

3.5.3 Search criterion over the dataset using the ABI model

Using the ABI model in iris code retrieval operation will enhance the search operation especially for large datasets. After applying the ABI iris code representation algorithm, we have got the A, B and I arrays and P which presents the number of pointers that goes from array A to array B, so the iris codes dataset will contain the A, B, I and number of pointers for each iris code.

Tables 5 and 6 present an example for the human iris codes dataset before and after applying the ABI model over the codes, for simplicity we examine iris codes of 24 bits.



	Human iris codes																						
0	0	1	0	1	1	1	1	1	0	1	0	0	0	1	0	1	0	0	0	1	1	1	0
1	0	0	0	0	1	1	1	1	1	0	0	0	1	0	1	1	0	1	0	1	0	0	0
0	0	0	1	1	1	1	1	0	1	0	0	1	1	1	0	0	1	0	1	0	1	0	0
1	1	1	0	0	1	0	1	0	1	1	1	0	0	1	1	0	0	0	1	1	0	1	1
0	1	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	0	0	1	0	1	0	1
1	0	1	0	1	0	1	0	1	1	1	1	1	0	0	1	1	1	0	0	0	1	1	1

Table 5: The human iris binary codes in the standard database.

Table 6: The representation of human iris binary codes in the ABI database.

																	H	Iun	nar	ı İri	is c	od	es													
			F	Դև	na	ry part Secondary part											Pointer index												No. of pointers							
0	P	1	1	P	P	0	P	P	0	1	P	*	1	*	*	1	1	*	1	1	*	*	1	1	4	5	7	8	11	*	*	*	*	*	*	6
0	P	1	1	P	0	1	P	P	P	P	0	*	0	*	*	0	*	*	1	0	0	0	*	1	4	7	8	9	10	*	*	*	*	*	*	6
1	P	P	P	P	1	0	1	0	P	P	1	*	1	0	0	0	*	*	*	*	0	1	*	1	2	3	4	9	10	*	*	*	*	*	*	6
Р	0	P	1	1	0	P	P	P	P	P	0	1	*	0	*	*	*	0	0	1	1	1	*	0	2	б	7	8	9	10	*	*	*	*	*	7
Р	P	P	P	1	1	P	P	1	0	P	1	1	1	1	1	*	*	1	0	*	*	0	*	0	1	2	3	б	7	10	*	*	*	*	*	7
Р	P	1	P	P	P	P	P	0	P	P	P	0	1	*	0	0	0	0	0	*	0	0	0	0	1	3	4	5	б	7	9	10	11	*	*	10

Note that the number of pointers in the iris code will be the base of the ABI search criterion over the dataset. Figure 3.6 illustrates the ABI search criterion over the database flow chart.





Figure 3.6: The ABI search criteria over the database flow chart.



The ABI search criterion algorithm initializes two parameters that act as counters that will count starting from the number of pointers of the input iris code to the number of pointers of the input iris code plus minus the C factor. Table 7 presents a description for these parameters.

Table 7: The parameters that configured by the ABI search criteria algorithm.

Description	Parameter
The counter that will count down from the number of	
pointers of the input iris code till the number of pointers	n1
minus the C factor.	
The counter that will count from the number of pointers of	
the input iris code up to the number of pointers plus the C	n2
factor.	

2- The ABI search criterion over the database stage

The basic idea for the ABI search criterion over the database is to benefit from the number of pointers of the input human iris code and the C factor. These information will help to calculate the maximum allowed differences in the number of the database iris codes pointers that will be matched with the input iris code.



So calculating the maximum difference will limit the number of the iris codes in the database that will be matched. This means that it is not necessary to visit all the database elements when applying the matching operation as is the case in the traditional iris codes retrieval.

After we apply the ABI iris code representation algorithm over the input iris code, we will start to compare it with only the iris codes of the dataset where the P value (the number of the database iris codes pointers) for them is in the range from P value of the input iris code \pm the C factor as mentioned in Equation (13).

$$Pdb \le Pin \pm C$$
 (13)

Where Pdb is the number of pointers from array A to array B of the database iris code, Pin is the number of pointers from array A to array B of the input iris code and C is the C factor that can be calculated using Equation (12).

After calculating the Pdb using Equation (13), the matching operation will be applied first over the iris codes with pointers equal to the input iris code pointers. Then the n1 and n2 will be decreased and increased by one respectively. Such that the direct of the matching operation will be from the database iris codes with nearest P to the P of the input iris code to the farthest ones.



This operation will be repeated until a match happened or n1 and n2 values exceed the value of Pin \pm C factor. So when using the ABI model, the database size will not affect the search criterion even if the database size is large.



4. DISCUSSION AND MATHMATICAL ANALYSIS



4 Discussion and Mathematical Analyses

In this section we presented an evaluation for the new ABI algorithm. The evaluation is based on the ABI algorithm mathematical analysis. Comparison of the ABI and the traditional iris code recognition models complexities. The test cases that present the headlines of all the cases that maybe appears during the human iris code recognition technique in both the standard and the ABI methods.

The entire subsections emphasized the necessity of this work and how well it improves the performance of existing system.

4.1 Mathematical Analysis

In this subsection we will study the complexity of the new ABI model aside with the standard iris code recognition model. The models complexities will provide a clear indicator to the enhancements that were made over the standard model via the ABI model.

4.1.1 The standard human iris recognition model complexity:

We can find clearly that the standard iris code recognition technique is based on two main operations. First the bit by bit comparison operation that is done over the input iris code and the iris codes in the database. Second is the linear search which presents the search criterion over the database.



Figure 4.1 presents the standard iris code recognition technique flow chart that accompanies the two basic operations. Note that the standard technique use two counters. The first one is used to count the database elements during the linear search operation (c). The second one is used to count the bits inside the iris code it self during the bit by bit comparison operation (d).

The standard human iris recognition model starts the matching operation with the first iris code in the data base. The matching operation compares each bit in the input iris code with the corresponding bit in the database iris code (this operation is repeated for the 2048 bits). If the resulted hamming distance does not exceed the specified threshold (HD \leq 0.32) then a match occurs. Else if (HD > 0.32) then the matching operation will be applied with the next iris code in the data base.





Figure 4.1: The standard human iris recognition technique flow chart.



In order to calculate the complexity of any algorithm, we need to know its basic operations and calculate the complexity for each operation separately which will help to calculate the algorithm total complexity. Table 8 illustrates the standard algorithm basic operations and their complexities.

Basic OperationOperation ComplexityThe bit by bit comparisonM, where M is the length ofoperation at the iris code levelthe iris codeThe linear search operation atN, where N is the number ofthe DB levelthe iris codes in the database

Table 8: the standard human iris recognition algorithm basic operations and their complexities.

For the searching algorithms the best case in general is to find the needed pattern from the first time, this is also applied on the standard human iris recognition algorithm. So the best case for the standard algorithm can be reached when a match occurs between the input human iris code and the first human iris code in the database. In this case only the bit by bit operation will be taken into account to calculate the best case complexity. The standard human iris code recognition best case is shown in Equation (14).

$$B(S) = M \tag{14}$$

Where B(S) is the standard human iris code recognition best case and M is the number of bits in the iris code.



As we know for all the searching algorithms, if the needed pattern locate at the end or is not found in the searching area we will reach the worst case. For this if any of the two cases happened when applying the standard human iris code recognition algorithm the worst case will be reached.

In the worst case the bit by bit comparison operation at the iris code level will be taken into account aside with the linear search operation at the database level to calculate the complexity. The standard human iris code recognition worst case is shown in Equation (15).

$$W(S) = M * N \tag{15}$$

Where W(S) is the standard human iris code recognition worst case, M is the number of bits in the iris code and N is the number of iris patterns in the database.

4.1.2 The proposed ABI model complexity

In the previous chapter (THEORY AND IMPLEMTATION) we have mentioned a full discussion for the proposed ABI model, its basic steps (code representation, initial code matching, main code matching and the DB search criterion) and how it works starting from the human iris code entrance until a match occurs with an existing code in the DB.



It is clear from the previous section that there are two main steps for the ABI model. Firstly the human iris code representation that is applied only once (at the beginning of all the code matching trials). Secondly the DB searching operation until a match occurs (this step contains both the initial iris code matching and the main iris code matching stages which will be repeated every time when moving to the next code in the DB to apply the matching operation). Figure 4.2 presents the basic steps of the proposed ABI model.



Figure 4.2: The ABI model basic steps.

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The ABI model algorithm complexity is equal to the summation of the iris code representation step complexity and the DB searching criterion complexity. The DB searching criterion is determined by the summation of the initial iris code matching step and the main iris code matching step multiplies by the number of iris codes that were visited until a match occurs. Table 9 illustrates the ABI algorithm basic operations and their complexities.

Basic Operation	Operation Complexity				
The human iris code representation	M, where M represents the length of the iris				
step	code (the total number of iris code bits).				
The initial iris code matching					
operation	<i>1</i> , this is a simple subtraction operation.				
The main iris code matching	(M - C), where M is the total number of iris				
operation	code bits and C is the C factor.				
	\boldsymbol{R} , where \boldsymbol{R} is the number of iris codes in				
	the range $[P-C, P+C]$, where P is the				
The linear search operation at the	number of pointers in the represented iris				
DB level	code and <i>C</i> is the C factor. Note that R in				
	the worst case is equal to N (the total				
	number of the iris codes in the database).				

Table 9: The proposed ABI algorithm basic operations and their complexities.

As in any searching algorithm, the best case of the proposed ABI algorithm is to find the needed pattern from the first time, so the database searching operation complexity will not be included in the calculation of the ABI algorithm best case complexity.



For a match to occur between the input iris code and the first iris code in the database, first the iris code representation is applied for the input iris code. Then the initial iris code matching between the represented iris code and the first iris code in the database is implemented. And the resulted value of the IHD is less than 0.32. After that the main iris code matching operation is processed between the two codes until (M - C), at this stage a match occurs.

So the calculation of the best case complexity of the proposed ABI algorithm will be the summation of the iris code representation complexity, the initial iris code matching operation complexity and finally the main iris code matching operation complexity. The ABI algorithm best case is shown in Equation (16).

$$B(ABI) = M + (M - C) + 1$$
(16)

Where B(ABI) is the ABI algorithm best case complexity, which is a linear function. And M is the iris code representation complexity, 1 refers to the initial iris code matching operation complexity and (M-C) represents the complexity of the main iris code matching operation.

Note that the ABI best case is less than 2M. That means that the best case of the standard iris code recognition algorithm is better than the ABI algorithm only if a match



two algorithms best cases when a match occurs with the first 10 iris codes in the database.

The number of the iris code in the DB in which a match occurs with	The standard iris code recognition algorithm best case	The proposed ABI algorithm best case
1st iris code	М	M + (M-C) + 1
2nd iris code	2M	M + (M-C) + 2
3rd iris code	3M	M + (M-C) + 3
4th iris code	4M	M + (M-C) + 4
5th iris code	5M	M + (M-C) + 5
6th iris code	6M	M + (M-C) + 6
7th iris code	7M	M + (M-C) + 7
8th iris code	8M	M + (M-C) + 8
9th iris code	9M	M + (M-C) + 9
10th iris code	10M	M + (M-C) + 10

Table 10: The standard iris code recognition algorithm and the proposed ABI algorithm bestcases if a match occurs with any of the first 10 iris codes in the DB.

From Table 10 we can clearly notice that the disadvantage of the standard iris code recognition algorithm is that we have to visit all the iris code bits each time we move to the next iris code in the database. While using ABI model will reduce this operation to only once (at the beginning when applying the representation for the input iris code).

The ABI algorithm worst case is reached when the needed pattern is located at the end of the database or is not found. In the ABI algorithm there are two scenarios in which the worst case is happened in between, the best worse case BW(ABI) and the worst worst case WW(ABI).



In the ABI best worst case BW(ABI), the values of the IHD that was resulted from the initial iris code matching operation to all the previous iris codes in the database were greater than 0.32. This led to move to the next iris code in the database without applying the main iris code matching operation.

In this case the ABI best worst case complexity is calculated as a summation of the iris code representation complexity, the initial iris code matching complexity for all the previous iris codes, the main iris code matching complexity for the matched iris code and the complexity of the database search criterion. This is shown in Equation (17).

$$BW(ABI) = M + (M - C) + R$$
 (17)

Where BW(ABI) is the ABI algorithm best worst case complexity, which is a linear function, M is iris code representation complexity, (M-C) is the main iris code matching complexity for the matched iris code and R is the complexity of the database search criterion which is at most equal to N (total number of the iris codes in the database).

From Equation (17) we can clearly notice that the BW(ABI) is better than the W(S), since (M + (M-C) + R) is much less than (M * N) where R is at most equal to N.



While in the ABI worst worst case WW(ABI), each time we visit a new iris code in the database we apply a full cycle which means that the initial and the main iris code matching operations are applied. Also when applying the main iris code matching operation we can not decide that the two iris codes are not related to the same person until we reach to (M-C).

So the worst worst case complexity of the ABI will be calculated as a summation of the iris code representation and the searching criteria which include the initial and main iris code matching as mentioned in Equation (18).

$$WW(ABI) = M + R(M - C) + R$$
(18)

Where WW(ABI) is the ABI algorithm worst worst case complexity, which is a linear function, M is the number of bits in the iris code and R is the number of iris patterns in [P-C, P+C] which is at most equal to N (total number of the iris codes in the database).

In order to decide which algorithm worst case is better, Table 11 presents the worst case complexity for both algorithms for different values assigned to N and for the ABI algorithm we take the worst for R to be equal to N. note that the value of M is fixed to 2048 and so C factor is equal to 655.



The DB size (N)	Worst case for the standard algorithm W(S)	Worst worst case for the ABI algorithm WW(ABI)
1000	2048000	1396048
10000	20480000	13942048
100000	204800000	139402048
1000000	2048000000	1394002048
2000000	4096000000	2788002048
3000000	6144000000	4182002048
4000000	8192000000	5576002048
5000000	10240000000	6970002048
1000000	20480000000	13940002048

Table 11: The algorithms worst case with different database sizes.

It is clear from the results in Table 11 that the worst worst case of ABI algorithm is also better than the worst case of the standard iris recognition algorithm. Because the WW(ABI) is a linear function while the W(S) is a quadratic function. We can note also that as well as the database size increase the difference (gap) in the worst case for both algorithms is increasing.

Note that if we want to calculate the average case complexity or the complexity of any case, we have only to substitute the value of N (in the standard model) or R (in the ABI model) by the location of the needed iris code.

4.2 Test Cases

In this subsection we mentioned the cases (scenarios) that present the headlines of all the cases that maybe appear during the human iris code recognition technique depending on the ABI model algorithms (2 & 3). Note that we examine 40 bits iris codes for simplicity.



To classify these test cases we study the behavior of the real human iris binary codes from the CASIA database ver.3 (CASIA-IrisV3) by applying the ABI model over the iris codes. The ABI model implementation were done using java version 6.0 on 2.00 GHz, 2.00 GB of Ram laptop running under Windows VISTA.

4.2.1 Case 1

Here we discussed the case of deciding that the two iris codes refer to two different people from the initial iris code matching operation, in another words the calculated IHD is greater than 0.32.

Let the first iris code = 0100001010101010101000000101000001010 and the second iris code = 10101111101010000001010101011111010100, then the matching operation will take the following track:

Firstly we used algorithm1 to apply the codes representation operation, the output of algorithm 1 was as the following:

- The first code representation:



- The second code representation:



Secondly we used algorithm 2 to apply the iris codes pattern matching operation, in this case only the initial matching stage was applied.

- Applying the initial human iris code matching operation:

1- Calculate the difference; in this example I1 = $\{0, 3, 4, 5, 8, 9, 10, 14, 15, 18, 19\}$, I2 = $\{0, 1, 5, 6, 7, 11, 12, 13, 17, 18\}$ then using Equation (11) the difference = 15.

2- Calculate the initial HD using Equation (6):

In this case the IHD = 15 / 40 = 0.375.

This exceeds the threshold (HD \leq .32).

So from this stage we can decide that the given iris codes relate to two different persons without the need to apply the bit by bit comparison operation. While when using the standard method we have to visit every bit in both codes to calculate the hamming distance.



First iris code = 01000010101011110101000000101000001010 Second iris code = 10101111110101000000101010111111010100

Reference to Equation (6), $HD = 25 / 40 \rightarrow HD = .62$ which exceeds the threshold 0.32.

4.2.2 Case 2

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In this case we examined a two iris codes that have the IHD less than 0.32 and we can decide that they refer to two different persons when applying the main iris code matching operation only over the first half of the iris codes.

Also in this case during the bit by bit comparison operation in the main matching operation the access is only to array A which is considered as the best case of all the identical cases.

Let the first iris code = 0100001111111111010000000101000001011 and the second iris code = 111111010101000011101100000111100001110 then the ABI model is applied as the following:

Firstly we used algorithm1 to apply the codes representation operation, the output of algorithm 1 was as the following:

- The first code representation:



Secondly we used algorithm 2 to apply the iris codes pattern matching operation as the following:

- Applying the initial human iris code matching operation:

19} then using Equation (11) the difference = 11.

2- Calculate the initial HD using Equation (6):

In this case the HD = 11 / 40 = 0.275.


This does not exceed the threshold (HD \leq .32). So we have to apply the ABI Main stage.

- Applying the main human iris code matching operation:

1- Calculate the C factor using Equation (12): In this case the C factor = $40 * 0.32 \rightarrow C = 12$.

2- From the I arrays we can find that there are 4 differences in the second half (bits from 10 to 19) of the codes.

3- Apply the bit by bit comparisons over the first half of the codes (bits from 0 to 9): The number of differences in the first half = 10.

4- Here we have to ask if the total number of differences exceeds the C factor or not. In this example: 10 + 4 > 12.

At this step we can decide that the given iris codes refer to two different persons without the need to apply the bit by bit comparison operation for the rest of the codes. Note that during the bit by bit comparison operation in step 3 the access was only for the A array, this means that we access only (1/4) from the total iris code bits. If we apply the standard human iris code recognition method:

First iris code = **010000111111111101000100000101000001011** Second iris code = **11111101010100001110110000001111000001110**



The resulted HD = 19 / 40 = 0.475 which exceeds the threshold 0.32.

4.2.3 Case 3

In this case we examined a two iris codes that have the IHD less than 0.32 and we can decide that they refer to two different persons when applying the main iris code matching operation only over the first half of the iris codes.

Here during the bit by bit comparison operation the access will be to both A and B, in some comparisons the access is only to A without the need to access B and for another comparisons it is necessary to access B.

Let the first iris code = 0100001010101010101000000101000001010 and the second iris code = 101111010101000010101000000111100001110 then the ABI model is applied as the following:

Firstly we used algorithm1 to apply the codes representation operation, the output of algorithm 1 was as the following:

- The first code representation:



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- The second code representation:



Secondly we used algorithm 2 to apply the iris codes pattern matching operation as the following:

- Applying the initial human iris code matching operation:

1- Calculate the difference; in this example I1 = {0, 3, 4, 5, 8, 9, 10, 14, 15, 18, 19}, I2

 $= \{0, 3, 4, 5, 8, 9, 10, 19\}$ then using Equation (11) the difference = 3.

2- Calculate the initial HD using Equation (6):

In this case the IHD = 3 / 40 = 0.075.

This does not exceed the threshold (HD \leq 0.32). So we have to apply the ABI Main stage.

- Applying the main human iris code matching operation:

1- Calculate the C factor using Equation (12):

In this case the C factor = $40 * 0.32 \rightarrow C = 12$.



2- From the I arrays we can find that there are 3 differences in the second half of the codes.

3- Apply the bit by bit comparisons over the first half of the codes (bits from 0 to 9): The number of differences in the first half = 10.

4- Here we have to ask if the total number of differences exceeds the C factor or not. In this example: 10 + 3 > 12.

At this step we can decide that the given iris codes refer to two different persons without the need to apply the bit by bit comparison operation for the rest of the codes. If we apply the standard human iris code recognition operation:

First iris code = 01000010101011110101000000101000001010 Second iris code = 1011110101010000101010000000111100001110

The resulted HD = 26 / 40 = 0.65 which exceeds the threshold 0.32.

4.2.4 Case 4

In this case we examined a two iris codes that are identical, this time when applying the bit by bit comparison operation we only visit bits in A array which considered the best case of the identical cases.



Firstly we used algorithm1 to apply the codes representation operation, the output of algorithm 1 was as the following:

- The first code representation:

Array A:																					
	0	0	0	1	1	1	1	0	0	0	1	1	0	0	1	0	1	1	1	1	
Array B:																					
-																					
Array I:																					

- The second code representation:

Array A:																						
	1	0	0	1	L	1	1	1	0	0	1	1	1	()	0	0	0	1	1	1	0
Array B:																						
Array I:																						



Secondly we used algorithm 2 to apply the iris codes pattern matching operation as the following:

- Applying the initial human iris code matching operation:

1- Calculate the difference, in this example $I1 = \Phi$, $I2 = \Phi$ then using Equation (11) the difference = 0.

2- Calculate the initial HD using Equation (6):

In this case the IHD = 0 / 40 = 0.0.

This does not exceed the threshold (HD \leq 0.32).

Note that to get an IHD = 0 does not mean that the two iris codes are identical. So we have to apply the ABI Main stage.

- Applying the main human iris code matching operation:

1- Calculate the C factor using Equation (3):

In this case the C factor = $40 * .32 \rightarrow C = 12$.

2- From I arrays we can find that there are 0 differences in the second half of the codes (bits from 10 to 19).



3- Apply the bit by bit comparisons over the first half of the codes (bits from 0 to 9). The number of differences in the first half = 2.

4- Here we have to ask if the total number of differences exceeds the C factor or not. In this example: 2 + 0 < 12.

5- In this case we have to check if the number of differences in the first half plus all the remained bits in the second half exceeds the C factor or not. In this example: 2 + 10 = 12.

From here we can decide that the two iris codes are identical without the need to compare the remaining bits. If we apply the traditional search operation:

The resulted HD = 8 / 40 = 0.20 which does not exceed the threshold 0.32.

4.2.5 Case 5

In this case a two identical iris code is examined; here both arrays A and B will be accessed during the bit by bit comparison operation (some times we will not need to access array B).



Let the first iris code = 0100001010101010101000000101000001010 and the second iris code = 01111101100111110101000000101000001111 then the ABI model will be applied as follows.

Firstly we used algorithm1 to apply the codes representation operation, the output of algorithm 1 was as the following:

- The first code representation:



- The second code representation:



Secondly we used algorithm 2 to apply the iris codes pattern matching operation as the following:

- Applying the initial human iris code matching operation:

1- Calculate the difference; in this example $I1 = \{0, 3, 4, 5, 8, 9, 10, 14, 15, 18, 19\}$, $I2 = \{0, 3, 4, 5, 8, 9, 10, 14, 15\}$ then using Equation (11) the difference = 2.

2- Calculate the initial HD using Equation (6):

In this case the IHD = 2/40 = 0.05.

This does not exceed the threshold (HD \leq .32).

- Applying the main human iris code matching operation:

1- Calculate the C factor using Equation (3):

In this case the C factor = $40 * .32 \rightarrow C = 12$.

2- From I arrays we can find that there are 2 differences in the second half of the codes (bits from 10 to 19).

3- Apply a bit by bit comparison over the first half of the codes (bits from 0 to 9): The number of differences in the first half = 4.



4- Here we have to ask if the total number of differences exceeds the C factor or not. In this example: 4 + 2 < 12.

5- In this case we have to check if the number of differences in the first half plus all the remained bits in the second half exceeds the C factor or not. In this example: 4 + 10 > 12.

So we have to repeat the previous steps but this time we will fragment the second half.

1- From I arrays we can find that there are 2 differences in the new second half of the codes (bits from 15 to 19).

2- Apply a bit by bit comparison over the new first half of the codes (bits from 10 to 14):

The number of differences in the first half = 0.

3- Here we have to ask if the total number of differences exceeds the C factor or not. In this example: 4 + 0 + 2 < 12.

4- In this case we have to check if the number of differences in the first half plus all the remained bits in the second half exceeds the C factor or not.

In this example: 4 + 0 + 5 < 12.



From here we can decide that the two given codes refer to the same person without the need to continue and compare the remained bits, because if all the remained bits were different the resulted differences will not exceed the C factor. If we apply the standard approach:

First iris code = 01000010101011110101000000101000001010 Second iris code = 01111101100111110101000000101000001111

The resulted HD = 10 / 40 = .25 which does not exceed the threshold 0.32.

4.2.6 Case 6

In this case we examined a two iris codes that are identical but each time we apply the bit by bit comparison operation we have to visit B array (this means that we will visit A array and then via pointer we will be forced to visit the B array).

Let the first iris code = 010101101010101010101010101010101010011001 and the second iris code = 0110011001010101010101010101010010001 then the ABI model is applied as the following:

Firstly we used algorithm1 to apply the codes representation operation, the output of algorithm 1 was as the following:



- The first code representation:



Secondly we used algorithm 2 to apply the iris codes pattern matching operation as the following:

- Applying the initial human iris code matching operation:

1- Calculate the difference; in this example I1 = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19}, I2 = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19}then using Equation (11) the difference = 0.



2- Calculate the initial HD using Equation (6):

In this case the HD = 0 / 40 = 0.0.

This does not exceed the threshold (HD \leq .32).

Note that to get an IHD = 0 does not mean that the two iris codes are identical. So we have to apply the ABI Main stage.

- Applying the main human iris code matching operation:

1- Calculate the C factor using Equation (3):

In this case the C factor = $40 * .32 \rightarrow C = 12$.

2- From I arrays we can find that there are 0 differences in the second half of the codes (bits from 10 to 19).

3- Apply the bit by bit comparisons over the first half of the codes (bits from 0 to 9): The number of differences in the first half = 3.

4- Here we have to ask if the total number of differences exceeds the C factor or not. In this example: 3 + 0 < 12.

5- In this case we have to check if the number of differences in the first half plus all the remained bits in the second half exceeds the C factor or not.

In this example: 3 + 10 > 12.



So we have to repeat the previous steps but this time we will fragment the second half.

1- From the I arrays we can find that there are 0 differences in the new second half of the codes (bits from 15 to 19).

2- Apply the bit by bit comparisons over the new first half of the codes (bits from 10 to 14).

The number of differences in the first half = 0.

3- Here we have to ask if the total number of differences exceeds the C factor or not. In this example: 3 + 0 + 0 < 12.

4- In this case we have to check if the number of differences in the first half plus all the remained bits in the second half exceeds the C factor or not.

In this example: 3 + 5 < 12.

From here we can decide that the two given codes are identical without the need to continue and compare the remained bits, because if all the remained bits were different the resulted differences will not exceed the C factor.



4.3 Test Cases analysis

From the mentioned cases we can conclude the following results:

4.3.1- For the different human iris codes

Case 1 presents the best case since we can decide that the two iris codes are different without the need to apply the bit by bit comparison operation (complexity equals to 1).

Case 2 presents the next best case since when applying the bit by bit comparison operation we only access the A array, which means that we apply the comparison operation on only 1/4 from the total number of iris codes bits.

The worst case is to reach M - C after accessing array B each time and then find that the two iris codes are different. Note that all the other cases will lie between the best and worst cases that were mentioned previously.

4.3.2- For the identical human iris codes:

Case 4 presents the best case since we have to visit only 1/4 from the total iris code bits to decide that the two iris codes are identical.

Case 6 presents the worst case, since we have to reach the M - C to decide the symmetry of the given codes also we forced to visit the bits in B each time we apply the comparison.



All the matching operations that are applied over the identical iris codes are located between the best and worst cases.

4.4 Hypothetical Case that can not be found in reality

In this subsection we examined a hypothetical case that can not be exist in reality according to the iris codes properties.

In this case the ABI behaves the same as the standard human iris code recognition technique; this means that we will apply the bit by bit comparison operation over all the iris codes bits.

The following are head lines about this case:

1. The IHD equals to 0.0 this means that we do not benefit from the initial matching operation.

2. We will not benefit from the differences of pointes in array I.

3. During the bit by bit comparison operation the access will always be to both arrays A and B (every time we have to access the array B).



Firstly we used algorithm1 to apply the codes representation operation, the output of algorithm 1 was as the following:

- The first code representation:



- The second code representation:



Secondly we used algorithm 2 to apply the iris codes pattern matching operation as the following:



- Applying the initial human iris code matching operation:

1- Calculate the difference using (11):

In this example the difference = 0.

2- Calculate the initial HD using (6):

In this case the IHD = 0.0.

This does not exceed the threshold (HD \leq .32). Not that to get an IHD = 0 does not mean that the two iris code are identical. So we have to apply the ABI Main stage.

- Applying the main human iris code matching operation:

1- Calculate the C factor using (3): In this case the C factor = $40 * .32 \rightarrow C = 12$.

2- From I arrays we can find that there are 0 differences in the second half of the codes (bits from 10 to 19).

3- Apply a bit by bit comparison over the first half of the codes (bits from 0 to 9). The number of differences in the first half = 10.

4- Here we have to ask if the total number of differences exceeds the C factor or not. In this example: 10 + 0 < 12.



So we have to repeat the previous steps but this time we will fragment the second half.

1- From the I arrays we can find that there are 0 differences in the new second half of the codes (bits from 15 to 19).

2- Apply a bit by bit comparison over the new first half of the codes (bits from 10 to 14).

The number of differences in the first half = 1.

3- Here we have to ask if the total number of differences exceeds the C factor or not. In this example: 10 + 1 + 0 < 12.

4- In this case we have to check if the number of differences in the first half plus all the remained bits in the second half exceeds the C factor or not.

In this example: 11 + 5 > 12.

So we have to repeat the previous steps but this time we will fragment the second half.

1- From the I arrays we can find that there are 0 differences in the new second half of the codes (bits from 17 to 19).



2- Apply a bit by bit comparison over the new first half of the codes (bits from 15 to 16).

The number of differences in the first half = 1.

3- Here we have to ask if the total number of differences exceeds the C factor or not. In this example: 10 + 1 + 0 < 12.

4- In this case we have to check if the number of differences in the first half plus all the remained bits in the second half exceeds the C factor or not.

In this example: 11 + 5 > 12.

In this case we will repeat the scenario until we reach the last bit, which means that the ABI model in this case will behave the same as the standard human iris code recognition technique.

- Why this case is considered a hypothetical case?

In reality; the bits in the human iris code are distributed as regions of zeroes and ones, which means that it is impossible to find a code with distribution for all its bits like (01010101010101). Figure 4.3 presents a part of a real human iris code; we can clearly notice the distribution of the bits.



Figure 4.3: Part from a real human iris code that clearly refutes the hypothetical case.



5. CONCLUSIONS AND FUTURE WORK



5 Conclusions and Future Work

5.1 Conclusions

In this thesis we proposed an efficient alternative to the standard human iris recognition technique, the ABI Model. This model basically represents the human iris binary code and then use this representation efficiently to apply the pattern matching and searching operations over the database.

Using the proposed ABI model will enhance the exhaustive human iris recognition technique in two directions, first the bit by bit comparison operation during the matching operation will not be applied over all the human iris code bits; second using the ABI model will determine the range of the database codes to apply the search criteria over it.

The efficiency of the suggested solution is measured by applying a detailed mathematical analysis for both the standard and the alternative ABI model and then comparing the resulted complexities; also examine the main cases that occur when applying the pattern matching operation which considered as the basic ones that all the other cases are lying in between.



We noticed that the best achieved improvement results when all the iris code bits lying in the array A and a match occurs after applying the main matching operation only for the first half, at this case the complexity is 1/4 from the complexity of the standard iris code recognition technique.

The worst case achieved when all the iris code bits lying in the array B and reach to M - C to determine the codes symmetry. Also we examine the worst case which is never appear in reality that occur when all the bits are lying in B and we apply the bit by bit comparison operation until reach the last bit in the iris code.

Do not forget to mention that the standard human iris recognition technique is better than the suggested ABI model in one case. This case is when a match occurs from the first time, in this case the standard complexity is equal to the iris code representation step of the ABI model and the rest ABI steps will increase the complexity.

But what makes the ABI better than the standard is that when we apply the iris code representation step only once (we visit the whole iris code bits only at the beginning) while in the standard method we have to visit the whole iris code bits every time we move to the next iris code in the database.



5.2 Future Work

As a future work we will implement a simulator to evaluate the iris code retrieval time (calculate the needed time to apply the iris representation, matching and searching operations). And then compare the resulted time with the retrieval time of the standard method.

Compare our results (complexity and retrieval time) with other methods that enhanced the standard method even if the enhancements were on other iris recognition stage (e.g. localization or normalization stages).

Study the ability to apply the ABI model over other physiological and behavioral characteristics such as face, fingerprints, eye retina, signature, and voice that are converted to binary strings and are processed in the same way as the iris pattern.

We will try to enhance the ABI model to be considered as a pre-step to all the searching operations over binary strings. This will at least specify a range of the database to apply the searching operation over it. So the size of the database will not be considered in the searching operations.



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ملخصص

في السنوات القليلة الماضية ازداد الاهتمام بتمييز الأشخاص من خلال الصفات الفيزيائية و البيولوجية المميزة لكل شخص, تعد قزحية العين أحد أهم الأنماط المميزة فبللر غم من صغر حجمها و صعوبة التقاط الصور الرقمية لها ، فان لها ميزة كبيرة تمكنها من التمييز بين الأنماط المختلفة بدرجة عالية من الكفاءة لذلك يفضل استخدامها عندما تكون هناك حاجة كبيرة للبحث في قواعد بيانات كبيرة الحجم دون الحصول على أية نتائج خاطئة على الرغم من وجود عدد هائل من الاحتمالات

إن الطريقة المتبعة من قبل نظم تمييز الأنماط و المعتمدة على نماذج قزحية العين لتحديد هوية الأشخاص تعاني من نوع من البطء و عدم الفاعلية عندما تقوم بالبحث في قواعد بيانات كبيرة الحجم . فقد أظهرت التجارب أن الوقت اللازم لتنفيذ عمليات مقارنة لاسترجاع قزحية العين من قاعدة بيانات بيانات بحجم . بحجم 100,000,000 حوالي 1000 ثانية الأمر الذي يحد من استخدام هذا النمط في التطبيقات التي تحتاج إلى الاسترجاع السريع.

إن الهدف من هذا البحث هو تحسين كفاءة النظم المتوفرة لتمييز الأشخاص من خلال قزحية العين عن طريق إعادة تمثيل الشيفرة الرقمية لقزحية العين باستخدام المصفوفات و من ثم الاستعانة بالتمثيل الجديد لاسترجاع قزحية العين من قاعدة البيانات و لكن هذه المرة سيكون الاسترجاع محصور ضمن النماذج الأقرب تمثيلا للنموذج المدخل.



لقد تم إجراء دراسة مقارنة بين نظام الاسترجاع الحالي و النظام المقترح في هذا البحث تمثلت بإجراء تحليل رياضي لكلا النظامين ثم مقارنة النتائج التي أظهرت بوضوح التحسين الذي وفره النظام المقترح أثناء عملية الاسترجاع.



